# Exercise 31

Find the orthogonal trajectories of the family of curves. Use a graphing device to draw several members of each family on a common screen.

$$y = \frac{k}{x}$$

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**Solution**

To find the orthogonal trajectories, we have to solve for $y'(x)$, set $y'_\perp$ equal to the negative reciprocal, and then solve for $y_\perp$. Start by differentiating both sides of the given equation with respect to $x$.

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{k}{x}\right)$$
$$\frac{dy}{dx} = -\frac{k}{x^2}$$

Solve the original equation for $k$,

$$k = \frac{y^2}{x^3},$$

and plug the expression into the equation.

$$\frac{dy}{dx} = -\frac{xy}{x^2} = -\frac{y}{x}$$

Here is where we introduce $y_\perp$.

$$\frac{dy_\perp}{dx} = \frac{x}{y_\perp}$$

Since this equation is separable, we can solve for $y_\perp$ by bringing all terms with $y_\perp$ to the left and all constants and terms with $x$ to the right and then integrating both sides.

$$y_\perp \, dy_\perp = x \, dx$$
$$\int y_\perp \, dy_\perp = \int x \, dx$$
$$\frac{1}{2}y_\perp^2 = \frac{1}{2}x^2 + C$$

Multiply both sides by 2.

$$y_\perp^2 = x^2 + 2C$$

Take the square root of both sides. Let $A = 2C$.

$$y_\perp = \pm\sqrt{x^2 + A}$$

This is the family of curves orthogonal to $y = k/x$.

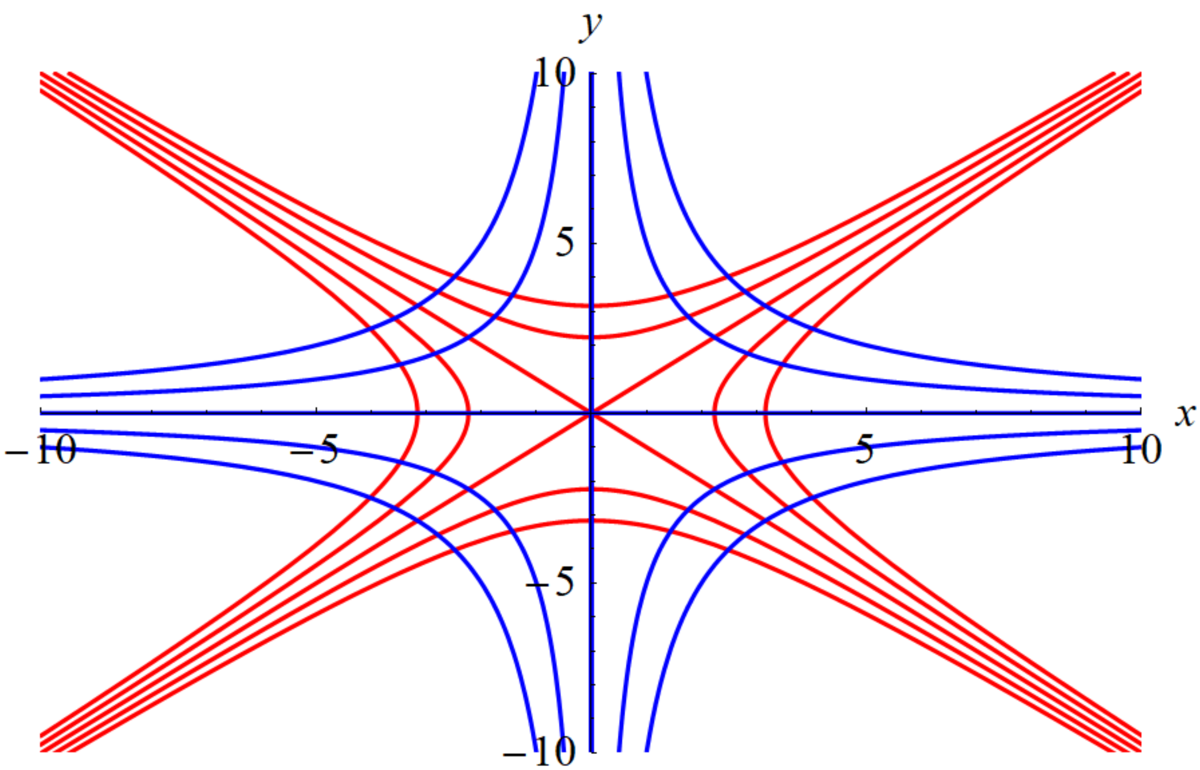


Figure 1: Plot of $y = k/x$ in blue ($k = 0, \pm 5, \pm 10$) and the orthogonal trajectories $y_\perp$ in red ($A = 0, \pm 5, \pm 10$).